NOTATION

R, radius of bubble; r, θ, spherical coordinates; v_r, v_θ, radial and longitudinal velocity components; z, vertical coordinate; U, velocity of rise of bubble; p_0, p_1, pressure; β_0, β_1, coefficient of volume expansion; ρ_0, ρ_1, density; μ_0, μ_1, dynamic viscosity; a_0, a_1, thermal diffusivity of liquid and vapor, respectively; σ, surface tension; $\tau_{r\theta}$, τ_{rr}, tangential and normal stresses; D, resisting force; $E(x, r)$, $F(x, r)$, elliptic integrals of the first and second kinds; $z = y/R$; $x = z \sin^2\theta$; $v_0 = v_{0,\theta}/U$; $v_1 = v_{1\theta}/U$; $F_i = v_i \sin\theta$; $\nu_i = \mu_i/\rho_i$; $C_D = 2D/(\pi\rho_0 U^2 R^2)$; $\mathrm{Re}_i = 2RU/\nu_i$; $\mathrm{Pr}_i = \nu_i/a_i$; $\mathrm{Pe}_i = \mathrm{Re}_i \mathrm{Pr}_i$; $\mathrm{Gr}_i = g\beta_i \times (dA/dz)R^4/\nu_i^2$; $\mathrm{Fr} = 8gR^3/\nu_0^2$; $\mathrm{Ma} = 2\frac{d\sigma}{dT}\frac{dA}{dz}R/\nu_0 a_0$; $\Phi(x) = \frac{2}{\sqrt{\pi}}\int_0^x \exp(-s^2)\,ds$; $\Phi^* = 1 - \Phi$; $i\Phi^*(x) = \int_x^\infty \Phi^*(s)\,ds$. Subscript $i = 0.1$.

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FREE CONVECTION IN A GRAINY LAYER ALONG A VERTICAL WALL

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A solution based on the integral thermal balance equation is offered.

We propose an approximate analytical solution of the problem of free convection produced by the temperature difference between a wall and a liquid filling an immobile grainy layer of solid elements. The solution obtained is also applicable to the process of mass exchange.

We make the following assumptions in considering the problem.

1. Liquid convection in the layer occurs in the region of dominance of viscosity forces.

2. The temperature difference in the layer is not large, so that the physical parameters of the liquid (aside from density) are temperature independent; the density is a linear function of temperature.

3. The temperatures of grains and liquid are identical, i.e., the layer is considered as a quasihomogeneous medium [1, p. 103].

4. Thermal conductivity in the layer along the liquid flow and thermal resistance at the wall [1, p. 127] are neglected.

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This last assumption will be evaluated below.

The heat-exchange process between the wall and the grainy layer is described by the equations

$$\rho c_p w_x \frac{\partial t}{\partial x} + \rho c_p w_y \frac{\partial t}{\partial y} = \frac{\partial}{\partial x}\left(\lambda_x \frac{\partial t}{\partial x}\right); \tag{1}$$

$$\frac{\partial w_x}{\partial x} + \frac{\partial w_y}{\partial y} = 0; \tag{2}$$

$$w_y = \frac{\Delta p}{l}\,\frac{C}{\mu} = g\beta(t - t_0)\,\frac{C}{\nu_0} \tag{3}$$

with boundary conditions:

$$\begin{aligned} &\text{at } x = 0,\ y > 0\ \ t = t_W,\ w_x = 0;\\ &\text{at } x \to \infty\ \ t \to t_0,\ w \to 0. \end{aligned} \tag{4}$$

For the effective thermal conductivity coefficient of the grainy layer, we will use the function [1, p. 112]

$$\lambda_x/\lambda_g = \lambda_0/\lambda_g + B_0 \operatorname{Re}\operatorname{Pr}, \tag{5}$$

in which the Reynolds number $\mathrm{Re} = w_y d/\nu$, B_0 is a proportionality coefficient of order of magnitude 0.1.

In dimensionless form, the problem appears as

$$W_x \frac{\partial \Theta}{\partial X} + W_y \frac{\partial \Theta}{\partial Y} = \frac{\partial}{\partial X}\left[(1/\mathrm{Ra} + B_0 W_y)\,\frac{\partial \Theta}{\partial X}\right]; \tag{6}$$

$$\frac{\partial W_x}{\partial X} + \frac{\partial W_y}{\partial Y} = 0; \tag{7}$$

$$W_y = \Theta \tag{8}$$

with boundary conditions

$$\begin{aligned} &\text{at } X = 0,\ \ Y > 0\ \ \Theta = 1,\ \ W_x = 0;\\ &\text{at } X \to \infty\ \ \Theta \to 0,\ W \to 0, \end{aligned} \tag{9}$$

where the following notation is used:

$$\begin{aligned} &X = x/d;\ Y = y/d;\ W = w/d;\ \Theta = \frac{t - t_0}{t_W - t_0};\\ &\mathrm{Ra} = \mathrm{Gr}\,\mathrm{Pr}\,\frac{C}{d^2}\,\frac{\lambda_T}{\lambda_0};\quad \mathrm{Gr} = \frac{g\beta(t_W - t_0)\,d^3}{\nu_0^2}. \end{aligned} \tag{10}$$

The value of w_W is defined by Eq. (3) at $t = t_W$: for a quasihomogeneous grainy layer the condition of adhesion of the liquid flow to the wall is not introduced, since all parameters in the layer are averaged over volumes larger than the volume of an individual grain. The ratio $\rho/\rho_0 \approx 1$ in accordance with assumption 2.

To obtain an approximate analytical solution we will use the integral thermal balance equation for the liquid flow near the wall

$$\left|\lambda_x \frac{dt}{dx}\right|_{x=0} = \frac{d}{dy}\int_0^\infty \rho c_p w_y (t - t_0)\,dx. \tag{11}$$

In dimensionless form

$$\mathrm{Nu}^*/Ra = (1/\mathrm{Ra} + B_0)\left|\frac{d\Theta}{dX}\right|_{X=0} = \frac{d}{dY}\int_0^\infty \Theta^2 dX, \tag{12}$$

where the modified Nusselt number $\mathrm{Nu}^* = \alpha d/\lambda_0$.

The form of the function which satisfies Eq. (12) and boundary conditions (9) may be found by analyzing Eq. (6). If we assume that in some region of space the terms on the left side of the equation are close to each other in magnitude, then for $1/\mathrm{Ra} \ll B_0$ we obtain the simple equation

$$\frac{d\Theta^2}{dY} = 0.5B_0 \frac{d^2\Theta^2}{dY^2}, \tag{13}$$

a solution of which is given by the Gaussian error function [2, p. 74]

$$\Theta^2 = \operatorname{erfc} z. \tag{14}$$

It is simple to prove that this function also satisfies Eq. (12), i.e., the desired solution has the form

$$\Theta = \left[\operatorname{erfc} \frac{X}{\sqrt{2(1/\mathrm{Ra} + B_0)Y}}\right]^{0.5}; \tag{15}$$

$$\mathrm{Nu}^* = \left(\frac{1/\mathrm{Ra} + B_0}{2\pi Y}\right)^{0.5} \mathrm{Ra}. \tag{16}$$

The mean value of the Nusselt number over length

$$\overline{\mathrm{Nu}}^* = \left(\frac{1/\mathrm{Ra} + B_0}{0.5\pi L}\right)^{0.5} \mathrm{Ra}; \tag{17}$$

$$K \equiv \overline{\mathrm{Nu}}^* \sqrt{0.5\pi L} = (1/\mathrm{Ra} + B_0)^{0.5}\,\mathrm{Ra}. \tag{18}$$

$$\text{for} \quad B_0\mathrm{Ra} \gg 1 \quad K = \sqrt{B_0}\,\mathrm{Ra}; \tag{19}$$

$$\text{for} \quad B_0\mathrm{Ra} \ll 1 \quad K = Ra^{0.5} \tag{20}$$

In the latter case the heat liberation coefficient α is independent of the characteristic dimension of the system d, inasmuch as it was assumed that the thermal conductivity of the layer is independent of the speed of liquid motion. For this case Taunton and Lightfoot [3] used numerical methods to obtain a solution of Eqs. (1)-(3) in the form

$$K = 1.11\,\mathrm{Ra}^{0.5} \tag{21}$$

Thus, the approximate solution of Eq. (20) differs from the exact one by 10%.

We will determine the limits of applicability of the formulas obtained. It follows from Eqs. (3) and (10) that

$$\mathrm{Re}_w \equiv w_w\, d/\nu = \mathrm{Gr}\frac{C}{d^2}; \quad \mathrm{Ra} = \mathrm{Re}_w \mathrm{Pr}\frac{\lambda g}{\lambda_0}. \tag{22}$$

A laminar flow regime exists in the layer for $\mathrm{Re} < 1$, but we may assume approximately that Eq. (3) is realistic up to $\mathrm{Re} = 10$ [1, p. 60]. Taking for a gas $\lambda_0/\lambda_g \approx 7$ [1, p. 107], we obtain the limiting value of the Rayleigh number $\mathrm{Ra}_l \approx 1$. For cold water $\mathrm{Ra}_l \approx 50$. For a laminar flow regime the value of the coefficient B_0 is approximately twice as small as for a turbulent regime [4], i.e., $B_0 \approx 0.05$, while for gases Eq. (20) is valid. For water, the convective component of the thermal conductivity coefficient in the layer may be comparable to the constant component λ_0 even in the laminar flow regime. For viscous liquids the value of Ra_l may be greater than 100, in which case Eq. (19) applies.

With regard to mass exchange, the value of the constant component of the diffusion coefficient is small, $D_0/D_g = 0.27$ [1, p. 89] and the convective component must be considered even for gases.

On the basis of the approximate solution obtained, Eq. (15), we will find the conditions under which thermal conductivity along the liquid flow need not be considered. To do this we compare the two terms of the differential equation

$$M = \frac{\partial}{\partial Y}\left[(1/\mathrm{Ra} + B_{0e}W_y)\frac{\partial\Theta}{\partial Y}\right]; \quad N = \frac{\partial}{\partial X}\left[(1/\mathrm{Ra} + B_0 W_y)\frac{\partial\Theta}{\partial X}\right].$$

For large values of Ra, calculations show a ratio for the region $\Theta > 0.1$

$$M/N < 2{,}4 B_{0e}/Y. \tag{23}$$

Longitudinal convective transfer in a laminar flow regime is characterized by the quantity $B_{0e} = 0.25$ [1, p. 100]; even at $Y > 10$ it may be neglected.

For small values of Ra in the region $\Theta > 0.25$

$$M/N < 8{,}5/\mathrm{Ra}\,Y. \tag{24}$$

A ratio $M/N < 0.1$ is acheived at $Y > 85/\mathrm{Ra}$. Thus, Eq. (20) is applicable only for sufficiently large L or at Ra not too far removed from unity. In the opposite case one must introduce into differential equation (6) the term M, not considered in the present solution or that of [3].

For heat liberation at the wall (within the limits of a hydrodynamic boundary layer at the wall) in [1, p. 138] formulas were presented which can be written in the parameters used herein as follows:

$$\overline{\mathrm{Nu}}_w^* \equiv \alpha_w d/\lambda_0 \approx 1{,}5 + 0{,}55\,\mathrm{Re}_w^{0.5}\,\mathrm{Pr}^{1/3} = 1{,}5 + 0{,}55\left(\mathrm{Ra}\,\frac{\lambda_g}{\lambda_0}\right)^{0.5}\mathrm{Pr}^{-1/6}. \tag{25}$$

For mass exchange, the constant component is absent from Eq. (25).

Calculations show that at $\mathrm{Ra} < 1$–2 and $L > 20$–30 the value of $\overline{\mathrm{Nu}}_w^*$ is an order of magnitude higher than $\overline{\mathrm{Nu}}^*$ and additional thermal resistance at the wall need not be considered. Resistance to mass transfer need not be considered at $L > 60$–100. For values of Ra and L of the order of 10^2 the value of $\overline{\mathrm{Nu}}_w^*$ becomes comparable to $\overline{\mathrm{Nu}}^*$.

In the general case, resistance to transfer at the wall can be considered approximately as

$$\overline{\mathrm{Nu}}_0^* = (1/\overline{\mathrm{Nu}}^* + 1/\overline{\mathrm{Nu}}_w^*)^{-1}. \tag{26}$$

In an exact solution of the problem it would be necessary to introduce a boundary condition of the third sort with consideration of Eq. (25).

In the formulas obtained the structure of the grainy layer was defined by the quantity C, the size of the elements by d, while the coefficient $B_0 = 0.05$ was taken for elements of round form with a layer porosity $\varepsilon = 0.4$. If the layer structure differs greatly from that indicated, or the layer is polydispersed, then it is desirable to choose as the characteristic dimension of the system, upon which the intensity of convective transfer in the flow depends, not d, but the equivalent diameter $d_e = 4\varepsilon/a$, where the surface of the elements a can be calculated from knowledge of their form and distribution over size [1, p. 14], or can be determined experimentally, simultaneously with the permeability [1, p. 50]. In this case the quantity B_0 is replaced by B/ε; the coefficient B may be considered independent of ε and close to B_0 for $\varepsilon = 0.4$ [1, pp. 95, 123].

NOTATION

a, grain surface in unit volume of layer; B_0, proportionality coefficient in Eq. (5); c_p, specific heat of liquid; C, layer permeability coefficient, m^2; d, mean grain diameter; l, wall height; t, temperature of grainy layer and liquid; t_0, same temperature far from wall; t_w, wall temperature; w, liquid filtration rate; x, coordinate perpendicular to wall; y, vertical coordinate; α, heat-exchange coefficient between wall and grainy layer; β, volume expansion coefficient of liquid; ε, layer porosity (fraction of voids); Δp, pressure drop in layer; λ, effective thermal conductivity coefficient in layer; λ_0, same coefficient for immobile liquid; λ_g, thermal conductivity coefficient of liquid (gas); μ, dynamic viscosity of liquid; ν, kinematic viscosity of liquid; ρ, density of liquid; ρ_0, density far from wall; Gr, Grashof number, referenced to d; Nu*, modified Nusselt number calculated from value of λ_0; Pr, Prandtl number for liquid at t_0; Ra, Rayleigh number, referenced to d; Re, Reynolds number for liquid flow in layer; X, Y, L, dimensionless counterparts of x, y, l, referenced to d.

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DROPLET BREAKUP REGIMES AND CRITERIA FOR THEIR EXISTENCE

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An analysis of experimental and theoretical studies of droplet breakup by a gas flow in shock tubes and nozzles is presented. A system of criteria defining droplet breakup regimes is developed.

When fuels are burned in various pieces of equipment it becomes necessary to analyze intermediate stages of preparation of the fuel mixture for ignition in the reaction zone. The processes of evaporation have been studied thoroughly and methods are available for their calculation, but the phenomenon of fuel atomization has been studied much less completely.

In the great majority of power devices production of useful work is accomplished by conversion of the chemical energy of the fuel into thermal energy, a process which usually occurs in a multiphase flow. The net velocity of the generally quite complex process of combustion of a liquid fuel is determined by the velocities of the elementary processes which occur: heating, evaporation, and atomization of liquid components, mixing and chemical reactions in the gas phase. For a proper description of the conversion process it is of extreme importance to know the principles by which these elementary processes operate.

One of the elementary acts having a great effect on the dynamics of the overall cycle is the process of droplet breakup in the gas flow. Acceleration of the two-phase flow, produced by geometric or thermal factors, leads to the appearance of a relative velocity between phases. Under the action of aerodynamic forces droplet deformation occurs, leading to droplet destruction [1-29]. The process of droplet and liquid jet breakup has been considered from such a position by many authors, beginning with Rayleigh [6, 8-11, 13, 21-23, 26-29]. In those studies it was shown that droplet and jet destruction occur under the condition that the Weber number exceed some critical value

$$W = \rho u^2 d/2\sigma > W^*, \tag{1}$$

where W* is the critical value of the Weber number.

It should be noted that the Weber number is not the unique criterion determining droplet behavior in a gas flow: depending on experimental conditions, its critical value varies over the range $W^* = 3$-25 [6, 10-12, 16, 21, 22]. In the most general case W* depends on the liquid viscosity [10], droplet diameter [7], and also upon the time for which the gas flow acts upon the droplet [14].

When Eq. (1) is satisfied in energy devices with intense heat supply, it is possible for secondary fuel droplet breakup to occur [1-3]. In determining the critical Weber number, until recently little attention was paid to how droplet breakup and deformation occur. At

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